Forty Years of Multiple Hypothesis Tracking

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Multiple hypothesis tracking (MHT) addresses difficult association problems in multiple target tracking by forming and evaluating data association hypotheses with multiple scans or frames of data. This paper reviews 40 years of MHT research and development since publication of the measurement-oriented MHT journal paper in 1979. It covers hypothesis-oriented and track-oriented MHT, distributed MHT, graph-based association, other MHT research, and the relationship with multitarget filters using random finite sets. It also reviews use of MHT in surveillance and other applications.

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I. INTRODUCTION

Data association is a key component of multiple target tracking (MTT) [1]–[10]. In fact, early papers [11], [12] in MTT frequently include "association" or "correlation" in their titles. The need to utilize multiple frames or scans of data for tracking multiple targets in difficult scenarios was recognized long ago, but early work focused on single target tracks, according to the survey in [1]. The use of multiple data association hypotheses to explain the origins of all measurements first appeared in the late 1970s, with batch solution of the best hypothesis by 0–1 integer programming [13], and recursive evaluation of multiple association hypotheses by computing their probabilities [14], [15]. Almost immediately, multiple hypothesis tracking (MHT) became the standard approach for tracking multiple targets when data association is difficult due to high target density, dense clutter, low probability of detection, etc.

Over the past 40 years, much research has been performed to generalize MHT [16] and address the inherent combinatorial growth in the number of hypotheses [17], [18]. The original measurement-oriented MHT, commonly called hypothesis-oriented MHT (HOMHT), is made practical by efficient algorithms to find the top-ranking hypotheses [19]–[23] and compute the bounds for the highest probabilities [24]. Track-oriented MHT (TOMHT) [17], [25]–[29] has been proposed as a more efficient alternative to the original HOMHT by maintaining association hypotheses at the individual track level and finding the best hypothesis only when needed, using integer programming, multidimensional (MD) assignment, or other methods [30]–[37]. Techniques for finding the top-ranking hypotheses are also available [38], [39].

When sensors are physically distributed, communicating measurements to a centralized tracker is often not feasible due to network bandwidth constraints. A distributed tracking system consists of trackers processing local sensor measurements and sending the results to another tracker for further processing. A distributed version of MHT that communicates hypotheses was proposed in [40]–[43]. Even though communicating hypotheses is not practical, this research identifies issues and techniques for associating tracks with dependent state estimation errors caused by prior communication or common process noise [44]. A more practical approach is communicating tracks from local trackers [45]. When MHT is performed on multiple platforms, the track pictures have to be consistent for distributed decision making [46]. For tracking with a single sensor, MHT is frequently used in a multistage architecture [47], with the first stage removing clutter to generate tracklets [48] of measurements that can be associated with individual targets without ambiguity, and the second stage associating tracklets [49].

Modern fusion systems utilize many sensors to track large numbers of targets. For large-scale tracking problems, even the most efficient TOMHT implementation suffers from combinatorial explosion. Association graphs have been proposed for implicit representation of all association ambiguities, with tracks represented by paths in the graph, and association hypotheses as sets of feasible paths [50]–[52]. When the track likelihoods satisfy a Markov property, a track likelihood is a product of pairwise association scores and the best hypothesis can be found by efficient graph algorithms [53]–[62]. However, MHT is most useful when the Markov property is not satisfied, e.g., when target feature data are present. Adapting graph-based algorithms for feature-aided association and non-Markov likelihoods is nontrivial [63]–[68].

Standard MHT assumes one-to-one association between measurements. Since this assumption is not valid for some tracking problems, MHT has been adapted to handle unresolved measurements [69], [70], multiple measurements from a single target [71], [72], extended objects [73], and merging and splitting targets [74], [75]. MHT requires data association to be consistent; i.e., measurements in a single scan/frame cannot be associated independently. Probabilistic MHT (PMHT) [76], [77] assumes independent measurement associations even though it has MHT in its name.

The computation complexity of MHT has resulted in much research to investigate other solution techniques. These include Markov chain Monte Carlo (MCMC) for data association [78]–[84] and message passing/belief propagation based on a graphical model of the tracking problem [85]–[88].

Displaying the output of MHT to an operator has to address track switching and jitter resulting from changes in the best or most likely hypothesis. Although MHT output display has not received as much attention as algorithm research, there is some progress in this area [89]–[92].

Detecting a target from a single frame of measurements is difficult when the signal-to-noise ratio is low. Using multiple frames to consider possible target trajectories can increase the probability of detecting targets and reduce false alarm rates. Multiple frame detection is basically track initiation or extraction and has been performed using MHT [93], [94], or with sequential probability ratio tests (SPRTs) [95]–[98].

Random finite set (RFS) for multitarget filtering has been a very active research area in recent years [99], [100]. Since the goal is finding the multitarget state probability density function (pdf), there is no explicit association in the model and filter equations. Thus, RFS-based filters appear to be different from MHT and cannot be used for tracking or forming trajectories, at least in the earlier forms [101]–[104]. Recent research has revealed MHT-like structures [105]–[114] in random set multitarget filters. In addition, MHT can be shown to have a solid theoretic foundation using random set formalisms [115], [116].

MHT is used primarily in defense and security applications where data association is difficult due to the nature of the targets. In particular, MHT is widely used in ocean, maritime, ground, air, and space surveillance [117]–[160]. Sensors include sonar, radar, electro-optical, seismic, etc. Each application domain has different target and sensor characteristics, resulting in different data association problems that MHT has to address. Because of the proliferation of video cameras, video tracking has become a very active area of research [161]–[188]. The nature of the association problem is amenable to efficient solution by graph-based methods. Other applications of MHT involve meteorology, astronomy, text messaging, cyber security, and biological and medical imaging [189]–[202].

This paper reviews key developments in MHT over the past 40 years. It may be viewed as a continuation of the tutorial in 2004 [17], and supplements the MHT chapters in books on tracking and fusion [2]–[10]. A review of this type reflects the limited knowledge and inevitable biases of the authors, especially given the large number of papers related to MHT published in diverse journals and conference proceedings. As of November 2019, [15] had 1530 citations according to IEEE Xplore and 3434 citations according to Google Scholar. Since we cannot include or read all references carefully, we apologize for omissions or misinterpretations and would appreciate any corrections or comments on this paper.

The structure of this paper is as follows. Section II presents target and sensor models, and defines hypotheses and tracks. Sections III and IV present the HOMHT and TOMHT. Section V discusses distributed MHT for single and multiple sensors. Section VI presents a graph model for data association and efficient solutions under the Markov assumption. Section VII discusses relaxation of assumptions and extensions to MHT. Section VIII presents the relationship between MHT and RFS approaches. Section IX lists some applications, and Section X concludes the paper by discussing possible research directions.

II. MULTIPLE HYPOTHESIS TRACKING

MHT uses target and sensor models to form association hypotheses for the origins of all measurements and computes their probabilities. We use MHT to stand for both multiple hypothesis tracking and multiple hypothesis tracker. The specific meaning should be obvious from the context.

A. Target and Measurement Models

The number of targets at time t is N_t. Each target has a hybrid (continuous–discrete-mixture-valued) state x_t. Given N_t, the target states are independent and identically distributed (i.i.d.) Markov processes with transition probability $f_{t|t'}(x|x')$.

Suppose there are K frames or scans of data taken at $t_1 \leq \cdots \leq t_K$. Each frame consists of m_k measurements $Z_k = (z_k^j)_{j=1}^{m_k}$, where a measurement z is independently generated from the target state x by the pdf $p_M(z|x)$ that may depend on time and the reporting sensor. A target at state x is independently detected according to a probability $p_D(x)$. For each k, the number of false alarms is N_{FAk} with some probability distribution density $p_{FA}(z)$ for their values.

B. Tracks and Hypotheses

A track τ is a sequence of measurement indices $(j_k)_{k=1}^K$ of cumulative measurements $Z_{1:K} \triangleq (Z_k)_{k=1}^K$ hypothesized to originate from the same target, where $j_k = 0$ indicates no measurement or that the target hypothesized by τ is undetected. A data association hypothesis λ is a collection of tracks that explains the origins of all measurements. In the MHT literature, it is common practice to refer to data association hypothesis as hypothesis. If the sensor resolution is such that two targets cannot generate one measurement, then two tracks in the same hypothesis cannot share the same measurement.

The definitions of track and hypothesis first appear in [13], which also views a hypothesis as a partition of the cumulative measurements. Data association hypothesis is sometimes called global hypothesis to distinguish it from track hypothesis that concerns only association of measurements with individual targets. We prefer to use global hypothesis to represent the association hypothesis that results from fusing local association hypotheses in distributed tracking.

III. HYPOTHESIS-ORIENTED MHT

HOMHT recursively generates hypotheses on the origins of measurements and computes the probability of each hypothesis. In the 1970s, there was a lot of interest in correlation techniques for naval ocean surveillance, where association with kinematic data only is difficult because observations or contacts are sparse. Thus, it is useful to use MHT to delay association decisions until good feature data are available. HOMHT consists of recursive generation, evaluation, and management of hypotheses [14], [15].

A. Hypothesis Generation

Let λ_{k-1} be a hypothesis on the cumulative data $Z_{1:k-1}$. Multiple new hypotheses λ_k on $Z_{1:k}$ are generated by hypothesizing different associations of the measurements in Z_k with the tracks in λ_{k-1}. A measurement z_k^j may be associated with an existing track τ_{k-1}^i in λ_{k-1}, with a newly detected target, or be hypothesized as a false alarm. A target hypothesized by an existing track τ_{k-1}^i in λ_{k-1}may not be detected in Z_k. This approach is called measurement oriented in [15] because it uses possible origins of measurements to generate new hypotheses. It is commonly called hypothesis-oriented MHT because of the recursive generation of hypotheses.

B. Hypothesis Evaluation

Let Z and $\bar{Z}$ represent $Z_{1:k}$ and $Z_{1:k-1}$, and λ and $\bar{\lambda}$ be hypotheses on Z and $\bar{Z}$ such that $\bar{\lambda}$ is the unique predecessor of λ. For a track τ in λ and any frame index $k' \in K$, let $Z_{k'|\tau}$ be the measurement in $Z_{k'}$ specified by τ. If τ has no measurement in $Z_{k'}$, then we say $Z_{k'|\tau} \triangleq \theta$, representing a hypothesized nondetection. Let $Z_{|\tau}$ be the sequence of measurements specified by τ. Then, the probability of the hypothesis is evaluated recursively by

$$P(\lambda|Z) = C(Z)^{-1} P(\bar{\lambda}|\bar{Z}) L_k^{FA}(Z_k|\lambda) \prod_{\tau \in \lambda} L_k(Z_{k|\tau}|\bar{Z}_{|\bar{\tau}}), \tag{1}$$

where $C(Z)$ is a normalization constant, $L_k^{FA}(Z_k|\lambda)$ is the likelihood of N_{FT} hypothesized false alarms given by

$$L_k^{FA}(Z_k|\lambda_k) = \beta_{FAk}^{N_{FT}} \tag{2}$$

with constant false alarm density β_{FAk}, and $L_k(Z_{k|\tau}|\bar{Z}_{|\bar{\tau}})$ is the likelihood of associating $Z_{k|\tau} = z_k^j$ with the predecessor track $\bar{\tau}$ in $\bar{\lambda}$. There are three types of $L_k(z_k^j|\bar{Z}_{|\bar{\tau}})$.

1) Likelihood of z_k^j from a previously detected target:

$$L_k(z_k^j|\bar{Z}_{|\bar{\tau}}) = p_{Dk} N(z_k^j - H_k \bar{x}_\tau, B_{k\tau}^j), \tag{3}$$

where $N(x, P)$ is the zero-mean normal density with covariance P, H_k is the measurement matrix, and $\bar{x}_\tau$ and $B_{k\tau}^j$ are the predicted estimate and corresponding error covariance, respectively.

2) Likelihood of previously detected target being undetected:

$$L(\theta|Z_{|\bar{\tau}}) = 1 - p_{Dk}. \tag{4}$$

3) Likelihood of the hypothesized number N_{NT} of newly detected target ($\bar{\tau} \triangleq \phi$):

$$L(z_k^j|Z_{|\phi}) = \beta_{NTk}. \tag{5}$$

Equations (1)–(5) define the algorithm in [14] and [15], using Poisson–Gaussian models for target dynamics and sensor measurements. Those likelihoods (2)–(5) are reformulated in [16] for more general target and sensor models, without linearity or Gaussian assumptions. When the number of targets is constant and Poisson distributed, the target states are i.i.d. random processes, and the number of false alarms is Poisson but not uniformly distributed, then the likelihoods are given by the following:

1) False alarm likelihood:

$$L_k^{FA}[Z_k|\lambda] = e^{-\bar{\nu}_{FAk}} \prod_{j \in J_{FAk}(\lambda)} \beta_{FAk}(z_k^j), \tag{6}$$

where $J_{FAk}(\lambda)$ is the set of measurement indices for the false alarms as hypothesized by λ, and $\bar{\nu}_{FAk} = \int_{E_M} \beta_{FAk}(z)\mu_M(dz)$ is the expected number of false alarms in frame k, with the measure μ_M on the measurement space.

2) Likelihood of z_k^j originating from a previously detected target ($\bar{\tau} \neq \phi$):

$$L_k(z_k^j|\bar{Z}_{|\bar{\tau}}) = \int_X p_{Mk}(z_k^j|x)p_{Dk}(x)p_k(x|\bar{Z}_{|\bar{\tau}})\mu(dx). \quad (7)$$

3) Likelihood of $\bar{\tau} \neq \phi$ being undetected:

$$L_k(\theta|\bar{Z}_{|\bar{\tau}}) = \int_X (1 - p_{Dk}(x))p_k(x|\bar{Z}_{|\bar{\tau}})\mu(dx). \quad (8)$$

4) Likelihood of z_k^j originating from a newly detected target:

$$L_k(z_k^j|\bar{Z}_{|\phi}) = \int_X p_{Mk}(z_k^j|x)p_{Dk}(x)\beta_{NT}(x)\mu(dx), \quad (9)$$

where $\beta_{NTk}(x) = \bar{\nu}_{NTk}p_k(x|\bar{Z}_{|\phi})$ is the density of undetected targets.

In (7) and (8), $p_k(x|Z_{|\tau})$ is the track state probability distribution determined by the predecessor track $\bar{\tau}$ and Z. When x is Gaussian, this distribution is represented by means and covariances. The hybrid measure μ is introduced to handle the hybrid state with both continuous and discrete variables. For discrete random variables, the integral becomes a summation.

The expected number ν_{NT} of targets that remain undetected through k frame is calculated from the expected number $\bar{\nu}_{NT}$ of undetected targets in $\bar{Z}$ as

$$\nu_{NTk} = \bar{\nu}_{NTk}\int_X [1 - p_{Dk}(x)]p_k(x|\bar{Z}_{|\phi})\mu(dx). \quad (10)$$

The state distributions for the tracks are updated by

$$p_k(x|Z_{|\tau}) = d^{-1}p_{Mk}(z_k^j|x)p_{Dk}(x)p_k(x|\bar{Z}_{|\bar{\tau}}) \quad (11)$$

for a track τ with a detection z_k^j, and

$$p_k(x|Z_{|\tau}) = d'^{-1}(1 - p_{Dk}(x))p_k(x|\bar{Z}_{|\bar{\tau}}) \quad (12)$$

for a track τ with no detection at frame k. Equation (12) is also used to compute $\beta_{NTk}(x)$, the density of undetected targets. In (11) and (12), d and d' are normalizing constants. The likelihoods (2)–(5) are a special case with linear and Gaussian models, and uniform detection probability.

C. Hypothesis Management or Implementation

Since the number of hypotheses grows rapidly with the number of frames, hypothesis management techniques are needed to make recursive MHT practical [15], [17]. Common techniques are pruning low-probability hypotheses, combining hypotheses with similar tracks, and decomposing targets and measurements into clusters [18] that can be solved independently.

Hypothesis pruning requires finding the hypotheses with the highest probabilities. At first, heuristic methods and search techniques were used, but results were frequently not satisfactory. HOMHT became practical only after efficient techniques for generating the m-best hypotheses were developed [19]–[21] using Murty's algorithm [22]. A reformation of the HOMHT [15] with Murty's algorithm is discussed in [23]. A method for estimating the bounds to the hypothesis probabilities is given in [24]. These bounds are useful for validating the correctness of the implementation.

When needed, the probability of a track can be computed as the sum of the probabilities of all hypotheses containing the track. Since it may be very difficult to enumerate and evaluate all hypotheses, track probability calculation is almost always approximate.

IV. TRACK-ORIENTED MHT

TOMHT is usually claimed to be more efficient than HOMHT because it recursively generates only tracks and finds the best association hypothesis only when needed [17], [25]. Even though [25] is one of the earliest references on the implementation of TOMHT, the concept of TOMHT first appeared in [13], which uses integer programming to find the best hypothesis over a batch of data, and a design for implementation is presented in [26].

A. Batch Hypothesis Evaluation

The probability of a hypothesis λ_K on the cumulative data $Z_{1:K}$ can be computed [40] as

$$P(\lambda_K|Z_{1:K}) = C(Z_{1:K})^{-1}l_K^{FA}(Z_{1:K}|\lambda_K)\prod_{\tau\in\lambda_K} l_K(\tau, Z_{1:K}), \quad (13)$$

where $C(Z_{1:K})$ is a normalizing constant, $l_K^{FA}(Z_{1:K}|\lambda_K)$ is the likelihood of false alarms, and $l_K(\tau, Z_{1:K})$ is the likelihood of the track τ given by

$$l_K(\tau, Z_{1:K}) = \bar{\nu}\prod_{k=1}^{K}\left\{\int g_k(Z_{k|\tau}|x)p_k(x|Z_{1:(k-1)|\tau})\mu(dx)\right\} \quad (14)$$

with the generalized likelihood $g_k(z|x)$ accounting for detection probability, i.e.,

$$g_k(z|x) = \begin{cases} p_{Mk}(z|x)p_{Dk}(x), & \text{if } z \neq \theta, \\ 1 - p_{Dk}(x), & \text{if } z = \theta. \end{cases} \quad (15)$$

Equation (13) assumes that the number of targets is Poisson distributed. Hypothesis evaluation for non-Poisson number of targets is discussed in [27]. This formulation assumes that there are no target births and deaths. Target appearance and disappearance are due to entry into and exit from the sensor field of view. Ref. [28] presents a model with target births that are never detected. The dimensionless scoring of MHT is discussed in [29].

B. Finding the Best Hypothesis

By suppressing the time index K, and using the appropriate normalization, (13) becomes

$$P(\lambda|Z) = C(Z)^{-1} \prod_{\tau \in \lambda} l(\tau), \tag{16}$$

where $C(Z)$ is a normalization constant. The best hypothesis is then the maximum a posteriori (MAP) solution for (16).

1) 0–1 Integer Linear Programming Formulation: Taking the negative logarithm of (16) and ignoring the normalization constant results in the following additive cost:

$$J(\lambda|Z) \triangleq \sum_{\tau \in \lambda} c(\tau), \tag{17}$$

where $c(\tau) = -\ln l(\tau)$. The optimization problem is then minimizing (17) subject to the constraint that λ does not have tracks sharing the same reports.

Let M be the number of tracks and $c = [c_1, ..., c_M]^T$ be the M-dimensional vector with $c_j = c(\tau_j)$. A hypothesis λ is represented by the M-dimensional vector $x = [x_1, ..., x_M]^T$, where $x_j = 1$ if track $\tau_j \in \lambda$, and $x_j = 0$ otherwise. Then, the MAP solution is given by the integer linear programming problem

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax \leq b \\ \text{and} & x_j \in \{0, 1\} \text{ for all } j \in \{1, ..., M\}, \end{array} \tag{18}$$

where A is an $N \times M$ matrix with $A_{ij} = 1$ if the report z_i is included in the track τ_j, and b is a vector of 1's with dimension N being the number of reports. The constraint $Ax \leq b$ states that tracks in a single hypothesis cannot share the same reports.

The integer linear programming formulation first appeared in [13], with an NP-hard exact solution. A solution is usually found by relaxing the integer value constraint of x_j and solving the standard linear programming problem [30]. When the solution is noninteger, branch-and-bound techniques are used.

2) MD Assignment Formulation: The MAP solution can be reformulated as an MD assignment problem [31]–[33]. Let j_k be the index of measurement $z_k^{j_k}$ in frame k and z_k^0 be a dummy measurement representing nondetection. A track hypothesis τ can be represented by an indicator function $\tau_{j_1 \ldots j_K}$, where

$$\tau_{j_1 \ldots j_K} = \begin{cases} 1, & \text{if } \tau = ((1, j_1), \ldots, (K, j_K)), \\ 0, & \text{otherwise}, \end{cases} \tag{19}$$

and a measurement (index) not in any track is a false alarm.

Let $c_{j_1 \ldots j_K} = c(\tau)$ be the cost of the track τ. Then, the minimization of (17) is equivalent to the following MD assignment problem:

$$\text{minimize} \quad \sum_{j_1=0}^{m_1} \cdots \sum_{j_K=0}^{m_K} c_{j_1 \ldots j_K} \tau_{j_1 \ldots j_K} \tag{20}$$

$$\text{subject to} \sum_{j_1=0}^{m_1} \cdots \sum_{j_{k-1}=0}^{m_{k-1}} \sum_{j_{k+1}=0}^{m_{k+1}} \cdots \sum_{j_K=0}^{m_K} \tau_{j_1 \ldots j_K} = 1 \tag{21}$$

for all $j_k = 1, 2, ..., m_k$ and $k = 1, 2, ..., K$. Constraints (21) specify that each measurement can belong to only one track. The cost $c_{00\ldots 0}$ is defined to be zero. If y is defined to be the vector formed from all $\tau_{j_1 \ldots j_K}$, then (20) and (21) have the form of minimize $c^T y$ subject to $By = 1$, which is an integer linear program.

Since the exact solution of MD assignment is NP-hard, approximate solutions are needed. Although there are differences in the specific steps, most approximate MD assignment techniques are based upon Lagrangian relaxation.

TOMHT can be formulated as the maximum weight independent set partition (MWISP) problem [34] with a hypothesis represented by a partition. This approach is not as popular as integer linear programming or MD assignment because the best partition is usually found by a greedy search procedure [35], [36].

C. Track Management or Implementation

Even though TOMHT is more efficient than HOMHT, the number of tracks still grows rapidly with the number of frames. Since the likelihood and state estimate must be generated for each track, efficient track management is essential. In addition to HOMHT hypothesis management techniques, N-scan pruning is a common method used in almost all TOMHT algorithms [17], [30]. After a best hypothesis is found, the tracks in the hypothesis are used to prune tracks that do not share ancestor nodes with them. As in HOMHT, clustering decomposes the data association problem into independent problems. An approach for clustering for MD assignment is described in [37].

Unlike HOMHT, TOMHT does not require hypothesis evaluation at each frame. Still it is useful to estimate the probability of the best hypothesis. Techniques for finding the m-best hypotheses have been developed for both MD assignment [38] and integer programming algorithms [39]. The probabilities of the m-best hypotheses can be used to compute the probability of a track.

V. DISTRIBUTED MHT

When sensors are physically distributed, communicating all measurements to a central tracker is often not feasible due to bandwidth constraints. In a distributed tracking system, the local trackers process the local sensor measurements and send the processing results to be fused by another tracker. Even when the sensors are colocated, it is sometimes desirable for each sensor to have its own tracker to distribute and simplify processing, especially when the sensors are of different types, such as radar and electro-optical.

A. Distributed MHT for Multiple Sensors

Distributed tracking must address issues such as what information should be communicated between local trackers, and how trackers process results from other trackers. The first distributed MHT assumes local trackers communicate and fuse local hypotheses and tracks [40]–[43]. Although the approach was demonstrated on a small distributed sensor network and validated using real flight data, communicating multiple hypotheses is not practical because it requires more bandwidth than sending sensor measurements. However, this research addresses the key issues for distributed tracking, such as removal of redundant information in tracks and evaluation of track-to-track association likelihoods. Almost all practical distributed MHT communicate a single hypothesis consisting of high-quality tracks.

The potential of using MHT in a hierarchical tracking architecture was recognized in the early days of MHT. In fact, the ocean surveillance correlation problem that motivated MHT research involves contact reports that are outputs from other systems. If these reports can be converted into measurements with independent errors, then MHT can process them in the usual manner. Otherwise, some form of decorrelation is needed to remove this dependence. The All Source Track and Identity Fuser (ATIF) [30] uses MHT to fuse tracks from multiple sensors. The first version avoids the temporal correlation in the track reports by processing the measurements in the tracks instead of the state estimates. The second version decorrelates the tracks to form equivalent measurements with independent errors. Let $\hat{x}^{s,i}_{k_1|k_1}$, $P^{s,i}_{k_1|k_1}$, $\hat{x}^{s,i}_{k_2|k_2}$, and $P^{s,i}_{k_2|k_2}$ be the state estimates and error covariances of a track i by sensor s at times k_1 and k_2 with $k_2 > k_1$. Then, the equivalent measurement $y^{s,i}_{k_2}$ and its covariance $V^{s,i}_{k_2}$ are given by

$$(V^{s,j}_{k_2})^{-1} y_{k_2} = (P^{s,i}_{k_2|k_2})^{-1} \hat{x}^{s,i}_{k_2|k_2} - (P^{s,i}_{k_1|k_1})^{-1} \hat{x}^{s,i}_{k_1|k_1}, \quad (22)$$

$$(V^{s,j}_{k_2})^{-1} = (P^{s,i}_{k_2|k_2})^{-1} - (P^{s,i}_{k_1|k_1})^{-1}. \quad (23)$$

The equivalent measurement represents the new information contained in the measurements of the track between k_1 and k_2, and is called tracklet in [48]. In this paper, we will follow the more common definition of tracklet as a short track consisting of measurements from the same target. This equivalent measurement of (22) and (23) is only approximate when the target dynamics have nonzero process noise [44]. The distributed MHT in [45] uses equivalent measurements from passive and active sensors to score association hypotheses.

The concept of data frame or scan is essential to HOMHT or recursive MHT. Since tracklets are not defined at a single observation time, there is no obvious way of organizing them into frames or scans. Thus, TOMHT is more appropriate for processing tracklets [49]. In particular, TOMHT has a natural formulation as graph-based association discussed in Section VI.

Due to processing differences, communication delays, and failures, the MHT on multiple platforms may produce different results. Conflicting track pictures are problematic when they are used for distributed decision making. An approach for maintaining a single integrated air picture for multiple platforms is developed in [46].

B. Multistage MHT for Single Sensor

Multistage processing for single sensor tracking is basically a data compression technique with a front-end tracker that processes the sensor measurements to remove clutter and generate tracks as inputs for the back-end tracker. The back-end tracker usually does some pre-processing such as checking the quality of input tracks and breaking them if necessary [47].

When the front-end tracker generates pure tracklets with little association uncertainty, the inputs to the back-end tracker can be represented by an association graph [50]. Then, the MHT can be solved efficiently if some Markov assumptions are satisfied, as discussed in Section VI.

VI. GRAPH-BASED ASSOCIATION

Advances in sensing and communication technologies have resulted in surveillance systems with many sensors collecting data on large numbers of targets. For example, ground-based or airborne video sensors are used to track moving vehicles in urban environments. Tracking with kinematic measurements alone is difficult due to high target density, occlusion from buildings, and large amounts of measurements. Thus, target feature observations are needed for accurate association and sparse feature data necessitate the use of MHT to maintain multiple hypotheses until feature observations are received to select the correct hypothesis. The “big data” problem is usually addressed with a hierarchical architecture with sensors generating pure tracklets and a high-level tracker associating the tracklets to form target tracks. The MHT problem can then be represented as an association graph [50], which has efficient solutions under some assumptions [51], [52].

A. Association Graph

Representation of tracks as paths over a trellis first appears in [53] and an efficient solution is given in [54]. However, it did not receive much attention in the traditional tracking community until recently even though graph representation of data association is quite standard in video tracking (Section IX-E). The nodes of an association graph are sensor reports that may be individual measurements or tracklets (sequence of measurements associated with same target with high confidence).

Each node is associated with a probability distribution of the state given measurements in the tracklet.

An edge connects two nodes when the reports can be associated with the same target. Two temporally overlapping tracklets from the same sensor cannot be associated. The weight of the edge represents the likelihood of association. Since association is a bidirectional relationship, the association graph is in general undirected. If the tracklets are from the same sensor, the graph is directed with a direction defined by the start or end times of the nodes.

An association graph provides an efficient implicit representation of tracks and hypotheses in MHT. A track is a path in the track graph and an association hypothesis is a set of consistent tracks, where consistency means that no two tracks in a hypothesis can share a single report. If there are no false reports, a hypothesis is a partition of all the reports or a nonoverlapping path cover of all the nodes.

B. Solution for Markov Association Likelihoods

Let $\tau = (y_1, ..., y_k)$ be a track with a tracklet τ_i represented by its measurements y_i. The likelihood of the track τ is

$$l(\tau) = \gamma_S(y_1) p_E(y_k) \prod_{i=1}^{k-1} l(y_{i+1}) \prod_{i=1}^{k-1} l(y^i, y_{i+1}), \qquad (24)$$

where $p_E(y_k)$ is the probability of the track ending after y_k, $\gamma_S(y_1)$ depends on the density of the new report y_1, $y^i \triangleq (y_1, ..., y_i)$ is the partial track with reports up to y_i, $l(y_i)$ is the likelihood of y_i, and $l(y^i, y_{i+1})$ is the likelihood of associating y_{i+1} with y^i.

1) Markov Likelihoods: The association likelihood satisfies the Markov property if $p(y_{i+1}|y^i) = p(y_{i+1}|y_1, ..., y_i) = p(y_{i+1}|y_i)$. Then, (24) becomes

$$l(\tau) = \gamma_S(y_1) p_E(y_k) \prod_{i=1}^{k-1} l(y_{i+1}) \prod_{i=1}^{k-1} l(y_i, y_{i+1}). \qquad (25)$$

The likelihood of a track is now the product of pairwise association likelihoods given by (25). The Markov property is also called the path-independent property because the association of nodes with a path depends only on the last node in the path and is independent of the rest of the path.

The Markov or path-independent property implies $p(x|y_1, ..., y_i) = p(x|y_i)$; i.e., the previous reports in a track cannot improve the estimate based only on the current report. This is true in tracklet stitching problems with very accurate sensors and fast target dynamics relative to the length of the tracklet [55], e.g., video tracking. The Markov property is not satisfied when the previous reports can improve the estimate computed using only the current report. Examples include raw sensor measurements, feature data, and multisensor reports that can be fused to improve the state estimate.

2) Bipartite Matching Formulation: An association hypothesis λ on the track graph can be represented by $x_{ij} \in \{0, 1\}$, $i = 1, ..., N$, $j = 1, ..., N$, so that $x_{ij} = 1$ if the directed edge (y_i, y_j) is in λ and 0 otherwise. With the Markov assumption, taking the negative logarithm of (16) and ignoring the normalization constant, the cost function for the MAP solution becomes

$$J(x) = \sum_{(i,j)\in E} c_{ij} x_{ij}, \qquad (26)$$

where $c_{ij} = -\ln(l(y_i, y_j)/(\gamma_S(y_j) p_E(y_i)))$ and E is the set of edges.

The best hypothesis is obtained by finding $x_{ij} \in \{0, 1\}$ that minimizes (26) subject to the constraints that each node i can be associated with at most one node j. This is a bipartite matching or assignment problem. Many efficient algorithms [56] have been developed to find the best matching or assignment. In addition, the K-best solutions can be found by Murty's algorithm [22].

3) Minimum Cost Network Flow Formulation: The bipartite matching formulation can be converted into the minimum cost network flow (MCNF) formulation. In fact, the MCNF solution first appeared in [54] for finding the best hypothesis for a trellis with the nodes representing radar measurements. However, this approach was ignored for many years because the Markov property does not hold for problems of interest at that time.

This approach was rediscovered with video tracking, which frequently uses a two-level architecture. The first or low level processes video data to form tracklets and the second or high level stitches the tracklets across occlusions or confusions. The tracklet stitching problem usually uses a track graph representation. Since the likelihood of associating a video tracklet with a video track usually depends only on the last tracklet in the track, the Markov property is satisfied and MCNF or bipartite matching algorithms can be used to solve the problem.

Because of this nice computational property, the Markov property is sometimes assumed in problems where it is clearly not valid. For example, [57] uses it for tracking with radar measurements and proposes an iterative approach to improve the solutions generated under the Markov assumption. Another example is multiple sensor track stitching where the path independence assumption is not valid [58]. Graph-based tracking systems with the Markov assumption have been developed and tested on real data involving a graph with several hundred thousand nodes.

The Viterbi data association approach of [53] is further developed for tracking [59]–[61]. A comparison of Viterbi-based and multiple hypothesis-based track stitching is investigated in [62].

C. Solution for Non-Markov Likelihoods

MHT is most useful when the data from later scans can significantly change the track likelihoods and reduce the association ambiguity. This is clearly not possible with Markov association likelihoods. The performance of graph-based techniques for track stitching is analyzed in [63].

Graph-based solution for MHT is an active area of research because traditional MHT cannot handle the data volume of modern surveillance systems. While the graph is a good representation of the association problem, standard graph analytic solutions require restrictive assumptions such as the Markov property. Solution of association graphs that violate the Markov assumption is an active research area [52], [64]–[68].

VII. OTHER MHT RESEARCH

A. Relaxing Measurement to Target Association Assumptions

Standard MHT assumes that a measurement in a single frame/scan cannot originate from two targets, and a target can generate at most one measurement in a frame/scan. This assumption is violated when low sensor resolution results in unresolved measurements, or high sensor resolution results in multiple measurements per target.

1) Unresolved Measurements: One way to handle association of unresolved measurements with multiple tracks is by modeling the unresolved measurements. HOMHT is used in [69] to track closely spaced aircraft with measurements from acoustic sensors. Before measurements are associated with the predicted tracks, track merging hypotheses are formed. The likelihood of associating a measurement with two tracks is computed from the probabilities of track merging and detecting the merged track, and the likelihood of associating the measurement with the detected merged track. The probability of unresolved targets is also a key component in [70], which addresses multiple hypothesis track maintenance for targets flying in close formation.

Unresolved measurements introduce merged track or unresolved track hypothesis in addition to measurement to track association hypothesis. Sophisticated hypothesis management techniques are necessary to make MHT practical for unresolved measurements.

2) Multiple Measurements: The sensors in some tracking systems generate multiple measurements per target. One such sensor is the over-the-horizon radar (OTHR), which generates multiple detections arriving over different propagation paths from the same target. Another example is passive coherent localization, which uses a single receiver to detect multiple measurements bouncing off the target from multiple transmitters.

Different MHT modifications have been proposed to address the multiple measurement problem. In [71], multiple measurements are viewed as detections of different modes, with a different measurement equation for each mode. A multiple detection multiple hypothesis tracker (MD-MHT) is developed to associate the measurements and estimate the mode. The MD-MHT uses MD assignment and its performance is demonstrated for OTHR tracking.

Ref. [72] addresses multiple measurements that are modeled by the same measurement equation. The multiple measurement MHT is based on a generalization of TOMHT recursion to handle repeated measurements. Tracking of multiple extended objects by Poisson multi-Bernoulli mixture (PMBM) filter, which has an MHT-like structure (Section VIII-A.2), is discussed in [73].

3) Split and Merged Targets: The targets in some tracking problems may split and merge, resulting in split and merged measurements. An example is tracking clouds that merge and split. Handling target merge and split requires modification of the standard MHT. In addition to new target from birth or first detection, extension of existing track, and false alarms, [74] also considers track split and track merge as possible origins of measurements. A different approach is used in [75], which decomposes the target state into kinematic/attribute state and event. The possible events are birth, death, split, and merge. The MHT has two steps: generating event hypotheses and data association hypotheses.

4) Probabilistic MHT: MHT assumes that a target can generate at most one measurement per scan. PMHT [76] uses an assignment model that violates this assumption, and allows measurement/target association to be independent across measurements. With this assumption, the optimization problem is changed from a combinatorial problem requiring solution by integer programming or MD assignment to a continuous optimization problem that can be solved by expectation–maximization algorithms. Even though MHT is in its name, solving the data association problem is not the primary objective of PMHT, which also assumes the number of targets is known. The problems and some solutions of PMHT are discussed in [77].

B. MCMC Data Association

It is well known that finding the best hypothesis of TOMHT by 0–1 integer linear programming (18) or MD assignment (20), (21) is an NP-hard combinatorial problem. Since MCMC methods [78], [79] can provide polynomial time algorithms to solve the NP-hard problem with sufficient accuracy, it is natural for MCMC to be used for target tracking [80].

In [81] and [82], the MC transition is defined as a combination of five "moves": 1) birth/death, 2) split/merge, 3) extension/reduction, 4) track update, and 5) track switch. Simulation results show that performance is better than commonly used TOMHT algorithms. MCMC is used to solve a multiple-intelligence (multi-INT) surveillance problem with good reported performance [66].

In [83], three MCMC sampling designs, Metropolis sampling, Metropolis sampling with Boltzmann acceptance probability, and Metropolis–Hasting sampling, are directly applied to the 0–1 linear programming formulation (18) of TOMHT. The results in terms of convergence speed are not very impressive when compared with an open-source mixed-integer linear programming package combining the primal–dual methods with the backup branch-and-bound method. At the suggestion by the late Dr. Jean-Pierre Le Cadre, another sequential Monte Carlo method known as cross entropy method was proposed in [84]. However, no definitive conclusion on the performance improvement was obtained.

C. Graphical Models for Data Association

Graphical models [85] are efficient representations of joint probability distributions of many random variables by exploiting factorization such as Markov properties. Given such a representation, inference algorithms are used to compute probabilities of specific variables or maximize some probabilities. By representing the MTT with a graphical model, message passing or belief propagation techniques can be used to solve the data association problem.

Ref. [86] presents message passing algorithms for solving a class of MTT problems. Ref. [87] uses a factor graph to represent the TOMHT and variational message passing to estimate the track probabilities. Empirical evaluation shows that track probabilities computed through message passing compare favorably to those obtained by summation over the k-best hypotheses

Ref. [88] uses an MWISP formulation of TOMHT and represents it by a graphic model. Max-product belief propagation is then used to find the MAP solution.

D. MHT Output

Designing a good display for a tracking system is always difficult because the information to display depends on the information needs of the operator. Displaying MHT output is particularly challenging. There are two choices: displaying the best (MAP) hypothesis or a combination of hypotheses. Usually the best hypothesis is displayed because finding alternative hypotheses is not easy. However, the tracks in the best hypothesis may change abruptly as the MAP hypothesis changes. This hypothesis hopping is very disconcerting to the operator as it results in track switching and jitter.

One way to generate smooth track estimates is by retrodiction [89], [90]. By introducing a delay and using a "smoothed" hypothesis, the track estimates will have fewer discontinuities. The retrodiction approach is acceptable if the delay is small enough for the mission.

Another approach [91] finds the real-time display of the target state estimates without any delay by minimizing the mean optimal subpattern assignment metric, which is defined in terms of the optimal subpattern assignment metric [92]. Since the display involves multiple hypotheses, the target estimates will be smoother.

E. Multiple Frame Detection and Track Initiation/Extraction

Detecting a target from a single frame of measurements is difficult when the signal-to-noise ratio is low and clutter is high. Multiple frame detection considers multiple candidate trajectories over the multiple frames and selects the best trajectories to detect or extract a target track. Since MHT performs track initiation in addition to track maintenance, it is a natural approach for multiple frame detection. The performance of MHT for track initiation and extraction is assessed in [93] and [94], with probability of establishing a track and number of false tracks as performance metrics.

The detection of small moving objects in a sequence of images is addressed in [95] by multistage hypothesis testing, which is also abbreviated as MHT. To avoid the complexity of standard MHT for target tracking, a sequential probability ratio test (SPRT) [96] is used to sequentially compare the statistics of two decision thresholds. A similar approach is used in [97] to extract tracks of weak but well-separated targets from high interference. More recently, [98] derives a Bayesian SPRT with new target density for track initiation based on the original HOMHT [15] and compares its performance with the classical SPRT [97], both theoretically and with simulations.

VIII. RELATIONSHIP TO RFSs

Multitarget filtering using random set formalism has been a very active area of research in recent years [99], [100]. In the random set approach, both the multitarget state and measurements are modeled as random sets. More specifically, the multitarget state at time t_k is the random set $X_k = \{x_k^1, ..., x_k^{n_k}\}$ and the measurements, which may be vectors, are the set $Z_k = \{z_k^1, ..., z_k^{m_k}\}$. Although RFSs are basically finite point processes allowing repeated elements, the RFS formalism, which does not allow repeated elements, is much more popular than the finite point process formalism. An appropriate concept of a pdf of an RFS $X = \{x_1, ..., x_n\}$ is the nth-order Janossy measure density function

$$f(\{x_1, ..., x_n\}) = n!p(n)f_n(x_1, ..., x_n), \qquad (27)$$

where $p(n)$ is the probability distribution on the number of elements, and $f_n(x_1, ..., x_n)$ is the joint pdf given n elements, which is symmetric or permutable, i.e., for any permutation π on $\{1, ..., n\}$, $f_n(x_1, ..., x_n) = f_n(x_{\pi(1)}, ..., x_{\pi(n)})$.

Let $f_{k|k-1}(X_k|X_{k-1})$ be the RFS state transition pdf, and $f_M(Z_k|X_k)$ be the RFS measurement likelihood. Then, the multitarget filter can be expressed by the prediction step

$$f(\mathrm{X}_k|\mathrm{Z}_{1:k-1}) = \int f_{k|k-1}(\mathrm{X}_k|\mathrm{X}_{k-1}) f(\mathrm{X}_{k-1}|\mathrm{Z}_{1:k-1}) \delta \mathrm{X}_{k-1}, \tag{28}$$

where the integral in (28) is the set integral over the measurable space of all the finite sets, and the update step

$$f(\mathrm{X}_k|\mathrm{Z}_{1:k}) = [f(\mathrm{Z}_{1:k}|\mathrm{Z}_{1:(k-1)})]^{-1} f_M(\mathrm{Z}_k|\mathrm{X}_k) f(\mathrm{X}_k|\mathrm{Z}_{1:k-1}). \tag{29}$$

A. MHT-Like Structures in RFS Filters

Since explicit target trajectories or tracks are needed in many applications, there has been much research on deriving MHT from RFS. In particular, RFS filters are shown to have MHT-like structures.

1) Cardinalized Probability Hypothesis Density (CPHD) Filter: The Probability Hypothesis Density (PHD) is the density of the measure defined as the expected number of targets within any measurable set in the target state space. The PHD filter approximates the predicted RFS pdf by a Poisson RFS pdf, at each updating stage, while the CPHD filter approximates it by an i.i.d. cluster RFS, without the Poisson assumption on the number of targets. The PHD filter provides the best Poisson RFS approximation in a Kullback–Leibler divergence sense [101]. Similarly, the CPHD filter provides the best i.i.d. cluster RFS approximation in Kullback–Leibler divergence sense, as proved in [102] and [103].

Neither PHD nor CPHD provides state pdf for each target, and the single-target state pdf cannot be inferred from the peaks of the posterior PHD. There are attempts to relate PHD or CPHD to MHT. For example, [104] shows that the Gaussian mixture CPHD filter is equivalent to MHT for single targets. Each Gaussian component has a predecessor and the sequence of predecessors forms a track.

2) PMBM Filters: A Bernoulli RFS is specified by the probability of existence for an element and a "spatial" pdf for the element if it exists. A multi-Bernoulli RFS is the union of independent Bernoulli RFS components.

The multi-Bernoulli mixture in the PMBM filter represents the posterior density $f(\mathrm{X}_k|\mathrm{Z}_{1:k})$ over the targets that have ever been detected, with each Bernoulli component representing a track and each mixture component representing a data association hypothesis. The addition of Poisson component represents targets that remain undetected. Thus, it is reasonable to expect that the PMBM filter will have structure similar to MHT [105].

In particular, [106] shows that the hypothesis evaluation equation (3) can be derived from the PMBM filter under the same assumptions of [15], including Gaussian–linear kinematics and no target death, and interpreting the new target density β_{NT} as the unknown target density in [105]. The derivation is based on representation by multi-Bernoulli mixtures. The relationship between HOMHT and PMBM filter is further analyzed in [107] by representing the multitarget pdf as a mixture of data association hypotheses that generalize the hypotheses of [15] by including undiscovered targets. In addition, new target density is viewed as a birth density. The resulting filter has essentially the same structure as the HOMHT, and the hypothesis probability recursion equation is (3) multiplied by a factor that represents undiscovered targets.

A Gaussian implementation of the PMBM is provided in [108], which also introduces the multi-Bernoulli mixture (MBM) filters. The difference between PMBM filters and MBM filters is that in PMBM filters the birth model is a Poisson point process, while in MBM filters, the birth model is multi-Bernoulli or multi-Bernoulli mixture. The prediction and update equations are analogous with a minor difference in the prediction step.

The PMBM filter does not establish explicit track continuity, which is desirable for MTT. By formulating the MTT problem as an RFS of trajectories [109], [110] derives PMBM trackers that estimate the trajectories, thus providing continuity and structure similar to MHT. Implementation of the trajectory PMBM filter is discussed in [111] and [112].

3) Labeled Multi-Bernoulli Filters: By adding a label to an individual target state, RFS-based filters explicitly maintain track continuity from entire trajectories of consecutive target states with the same label. If one labels the Bernoulli components in the MBM filter, which is a particular case of the unlabeled case, one gets a labeled MBM filter. If the labeled MBM filter is written with (data association) hypotheses in which target existence is deterministic rather than probabilistic, one gets the δ-generalized labeled multi-Bernoulli (δ-GLMB filter) [108, Sec. IV]. Having deterministic existence in each hypothesis implies an exponential increase in the number of hypotheses (and therefore in the number of data associations to be solved), and is thus not desirable.

δ-GLMB RFS filters [113], [114] have been used to develop multitarget tracking filters with structure similar to MHT. For example, the update equation for detected targets [86, eqs. (105) and (106)] can be used to illustrate the similarity to MHT. More specifically,

$$\begin{aligned} f(\mathrm{X}_k|\mathrm{Z}_{1:k}) &= \sum_{c_k} P(c_k) f_{c_k}^{\mathrm{LMB}}(\mathrm{X}_k) \\ &= \sum_{c_k} P(c_k) \prod_{l \in \mathrm{L}_k} f^{(l, c_k^l)}(\mathrm{X}_k^l), \end{aligned} \tag{30}$$

where L_k is the set of detected targets, X_k^l is $\{(x_k^i, l_k^i)\}$, $f^{(l,c_k^l)}(\mathrm{X}_k^l)$ is a Bernoulli pdf for each label l in L_k, and c_k is a data association vector with entries c_k^l, $l \in \mathrm{L}_k$, and probability $P(c_k)$. According to (30), the RFS pdf is the sum of LMB $f_{c_k}^{\mathrm{LMB}}(\mathrm{X}_k)$ representing the RFS pdf for each data association c_k, weighted by $P(c_k)$, and $f^{(l,c_k^l)}(\mathrm{X}_k^l)$ is the track RFS pdf.

B. RFS Formalisms for MHT

There are some concerns that MHT does not have a solid mathematical foundation because the derivations in [13]–[16] do not use complicated mathematics. In fact, there were even criticisms that MHT is heuristic. These concerns are addressed in [115] and [116], which introduce mathematical formalisms to support a theory for MHT.

In particular, [116] models the set of targets as (i) a random finite sequence, (ii) a finite point process, and (iii) a random finite set, of stochastic processes on the target state space over a given continuous-time interval $[t_0, \infty)$. Viewing an individual target as a stochastic process is similar to the use of target trajectories in [110].

Using the standard assumptions on the measurements, the hypothesis evaluation equation is

$$P(\lambda_K|Z_{1:K}) = P(Z_{1:K})^{-1} L_{\mathrm{FA}K}(\lambda_K) L_{\mathrm{NDT}K}(\#(\lambda_K)) \times \left(\prod_{\tau \in \lambda_K} L_K(\tau) \right), \tag{31}$$

where $L_K(\tau)$ is the likelihood of a track similar to (14). However, $L_{\mathrm{FA}K}(\lambda_K)$, the likelihood of false alarms, and $L_{\mathrm{NDT}K}(\#(\lambda_K))$, the likelihood of the cumulative number of detected targets, are more complicated unless the number of targets and number of false alarms satisfy the Poisson assumption. Note that the hypothesis evaluation equation (31) is the same for all three formalisms.

IX. APPLICATIONS

MHT has been applied to target tracking problems that require sophisticated methods for data association. Many of these applications are in defense and security, where government regulations and company policies restrict publications, especially on deployed systems. Our review will focus primarily on those published in the open literature. Applications of MHT for tracking ground targets, aircraft, and missiles are already discussed in [17]. We will discuss other applications such as ocean and maritime surveillance, space situational awareness, airborne video surveillance, video tracking, and some unconventional areas.

A. Ocean and Maritime Surveillance

1) Ocean Surveillance: Ocean surveillance is characterized by a huge surveillance region that may cover the entire world. Relative to the size of the surveillance region, there are few targets and they do not move very fast. The tracking problem would be easy except that few sensors provide persistent coverage, resulting in sparse measurements. Since kinematic measurements are not useful for association in many occasions, feature or attribute observations are valuable but not always available. The tracking problem is even more challenging for submarines because they are designed for stealthy operations.

Naval ocean surveillance was a very active area of research in 1979, with [117] documenting the state of the art around that time. Ref. [118] discusses using MHT of [15] for ocean surveillance with a target state that includes a continuous component representing position, velocity, and emitter parameters, and a discrete component representing attributes such as platform or radar identifications. An architecture for fusion of multisensor ocean surveillance data using MHT is proposed in [119].

Submarine tracking relies largely on acoustic sensors. A multiple hypothesis approach is proposed for concurrent mapping and localization for autonomous underwater vehicles [120]. The MHT approach for tracking with passive and active sonar is discussed in [121]–[124].

2) Maritime Surveillance: Maritime surveillance is characterized by a smaller surveillance region and many more sensors than ocean surveillance. However, there are more targets with high maneuverability. Maritime domain awareness requires tracking targets and monitoring their behaviors [125].

One system for port surveillance fuses video and radar data with automatic identification system (AIS) transponder data to form composite fused tracks for all vessels in and approaching the port using MHT. Rule-based and learning-based pattern recognition algorithms are then used to generate alerts [126], [127].

Ref. [128] discusses an MHT at the NATO Undersea Research Centre. An MHT that fuses radar and AIS data is described in [129]. While the targets of interest for ocean and maritime surveillance are surface and subsurface vessels, MHT has been used to estimate the number of beaked whales [130].

B. Ground Surveillance

Ground surveillance targets include vehicles, people, and animals, with movements in rural or urban environments. Sensors include radar, electro-optical, and others on airborne platforms, as well as ground-based sensors such as seismic.

1) Ground Moving Target Indicator: The utility of airborne ground moving target indicator (GMTI) radar for ground surveillance was demonstrated in the first Gulf War of 1991 [131]. The large amount of data produced by GMTI radar overwhelms manual analysis and requires automated tracking algorithms.

Ground target tracking is characterized by large number of targets that may be close to each other. The targets are highly maneuverable with move–stop–move-type behavior and on-road/off-road modes. Because of the observation geometry, the targets may be obscured by terrain. Furthermore, the MTI radar detects targets only when their radial velocities are above the minimum detectable velocity. Coverage gaps and highly maneuverable targets make data association difficult. The challenges of ground target tracking and available algorithms are discussed in [132].

Ref. [133] describes a U.S. program to develop GMTI tracking algorithms around 2000. The initial phase involved four contractors using HOMHT [134] and TOMHT. The two winning contractors later became the main tracking algorithm developers for ground surveillance in the United States. Other research in GMTI tracking includes [135]–[138].

2) Airborne Video: The targets for tracking with airborne video have similar characteristics to those in GMTI tracking. In addition, airborne video is often used to track people. Some airborne platforms such as the Predator have steerable sensors with narrow field of view, while other platforms have wide-area motion imagery (WAMI) sensors. Since steerable sensors have narrow field of view, accurate tracking is crucial to control the sensor to observe the targets. Thus, MHT is part of a closed-loop system with both tracking and sensor control [139], [140].

A WAMI sensor can detect many targets because of its wide field of view. When used for urban surveillance, occlusion from buildings and high target density makes data association very difficult. Furthermore, the goal of tracking is to produce tracks (trajectories), and not just estimating the locations, which would be very easy for video sensors. Thus, MHT is widely used for airborne video tracking. Approaches include TOMHT with integer programming [141]–[143], MD assignment [144], and graph-based approach [145], [146]. Graph-based approach is particularly applicable because the Markov assumption is valid.

C. Air and Missile Target Tracking

Air and missile targets have mostly well-defined motion models based on physics, even during maneuvers. Furthermore, sensors such as radar or infrared search and track do not have to contend with the occlusion problem in ground target tracking. However, military targets can fly in close formation, and are designed to escape detection with the help of countermeasures. These are challenges for air and missile target tracking.

The benefit of using MHT for sensor fusion in airborne surveillance systems was recognized very early [147], with performance assessments made in [93] and [94]. MHT algorithms are developed for infrared [148], electronically scanned radar [149], and multisensor air defense [150]. Interacting multiple models are used with MHT to handle target maneuvers [151], [152]. Targets flying in formations are addressed in [153]. More recent research uses MHT with active and passive sensors for the sense and avoid problem [45] and air surveillance system [154], extending earlier MHT work for air traffic control [155].

Missile defense is an important application for MHT [17] because of the high target density, difficult association problem for angle-only measurements from space base sensors, and the need for continuous birth to death tracking. However, very little research and development is reported in the open literature. An exception is boost-phase ballistic missile defense using MHT [156].

D. Space Situational Awareness

Space situational awareness (SSA) is important but difficult due to the large number of satellites and space debris in various orbit regimes. SSA has to discover new objects, catalog and track resident objects, and characterize tracked objects [157]. Since sensors observe space objects with large time gaps, data association is nontrivial. MHT has been recognized as a promising solution to the space object problem [158]–[160].

E. Video Tracking

Due to the availability of low-cost video cameras, video tracking has been a very active application area for computer vision researchers. The goal of video tracking is to maintain continuous tracks of targets, called objects in the video tracking community, and infer activities and intentions. Object tracking has to address abrupt object motion, changing appearance patterns, nonrigid structures for humans or animals, and occlusions. Association is usually more important than estimation because users can locate the objects on the image.

The association problem among image frames is called correspondence in the computer vision community. Due to object crossing, appearance change, and occlusion, correspondence using only two frames may result in incorrect correspondence. Better tracking results can be obtained if the correspondence is performed over several frames. Thus, MHT is a natural approach for solving the correspondence problem in video tracking.

A Bayesian multiple hypothesis approach for contour grouping, edge and contour segmentation [161], [162] leads to an efficient implementation of HOMHT for video tracking [163], based on finding ranked assignments [20], [21]. This is followed by other research on using MHT for video tracking [164]–[187]. Besides using standard MHT, e.g., [183], most research assumes the Markov assumption to build a track graph and uses efficient solutions such as MCNF. It is interesting to note that most of the research is performed in universities, unlike MHT applications in defense and security performed mostly in companies and government laboratories. The shifting of research to academia is due to easy access to data for algorithm development and testing from community datasets [188] and low-cost camera systems.

F. Other Nontraditional Applications

In addition to tracking traditional targets such as ships, vehicles, planes, and people, MHT has been used in other applications where good data association performance requires using multiple frames/scans of data. The following are some examples:

1) Ocean eddy current tracking [189].
2) Cloud tracking [74], [75].
3) Associating asteroid observations [190].
4) Solar events tracking [191], [192].
5) Tracking text messages and information [193], [194].
6) Detection of internet worms [195].
7) Cyberattack tracking [196].
8) Cellular traffic in living cells [197].
9) Tracking in biology and medical images [198]–[202].

X. CONCLUSION

We have reviewed the main research and development in MHT over the last 40 years. Since MHT is based on a sound mathematical formulation of a real MTT problem, research has focused on relaxing the assumptions of MHT, developing efficient implementations, and applying to problems that require association using multiple frames or scans of data. It is interesting to note that while almost all early research was performed in industry or government laboratories, most recent research is now performed in academia without a strong application focus. We believe future research should be driven by applications, since new problems may require relaxing other assumptions, which in turn requires new algorithms.

Most MHT algorithms assume targets have i.i.d. motion models. The independence assumption is violated when targets move as a group, or when vehicles are moving on a single-lane road. Exploiting this dependence should improve association performance.

MHT is designed to address difficult data association problems by maintaining association hypotheses over multiple frames of data. Experience with real data has shown that when data association is too easy, MHT is not needed, and when data association is too difficult, MHT does not help. Research is needed to predict when MHT is useful, which is related to computing the hypothesis probability distributions. Relating MHT to RFS may also result in algorithms that do explicit data association only when the data are good enough for association to make sense.

Some tracking problems cannot be solved without using sophisticated (or full-fledged) MHT. An example is tracking targets with frequent kinematic measurements and sparse feature observations. In such situations, MHT has to maintain multiple hypotheses until feature data are available for association. Efficient maintenance of hypotheses for long durations is an active area of research.

REFERENCES

1. Y. Bar-Shalom
"Tracking methods in a multitarget environment,"
IEEE Trans. Autom. Control, vol. 23, no. 4, pp. 618–626, Aug. 1978.
2. S. S. Blackman
Multiple-Target Tracking with Radar Applications. Norwood, MA: Artech House, 1986.
3. Y. Bar-Shalom and T. E. Fortmann
Tracking and Data Association. New York, NY: Academic Press, 1988.
4. Y. Bar-Shalom and X.-R. Li
Multitarget–Multisensor Tracking: Principles and Techniques. Storrs, CT: YBS Publishing, 1995.
5. S. S. Blackman and R. Popoli
Design and Analysis of Modern Tracking Systems. Norwood, MA: Artech House, 1999.
6. Y. Bar-Shalom, P. K. Willett, and X. Tian
Tracking and Data Fusion: A Handbook of Algorithms. Storrs, CT: YBS Publishing, 2011.
7. S. Challa, M. R. Morelande, D. Mušicki, and R. J. Evans
Fundamentals of Object Tracking. Cambridge, U.K.: Cambridge University Press, 2011.
8. L. D. Stone, R. L. Streit, T. L. Corwin, and K. L. Bell
Bayesian Multiple Target Tracking, 2nd ed. Norwood, MA: Artech House, 2013.
9. M. Mallick, V. Krishnamurthy, and B.-N. Vo
Integrated Tracking, Classification, and Sensor Management: Theory and Applications. Hoboken, NJ: Wiley, 2012.
10. C.-B. Chang and K.-P. Dunn
Applied State Estimation and Association. Cambridge, MA: MIT Press, 2016.
11. R. W. Sittler
"An optimal data association problem in surveillance theory,"
IEEE Trans. Mil. Electron., vol. 8, no. 2, pp. 125–139, Apr. 1964.
12. R. Singer and R. Sea
"New results in optimizing surveillance system tracking and data correlation performance in dense multitarget environments,"
IEEE Trans. Autom. Control, vol. 18, no. 6, pp. 571–582, Dec. 1973.
13. C. Morefield
"Application of 0–1 integer programming to multitarget tracking problems,"
IEEE Trans. Autom. Control, vol. 22, no. 3, pp. 302–312, Jun. 1977.
14. D. B. Reid
"An algorithm for tracking multiple targets,"
in *Proc. IEEE Conf. Decis. Control*, 1978, pp. 1202–1211.
15. D. B. Reid
"An algorithm for tracking multiple targets,"
IEEE Trans. Autom. Control, vol. 24, no. 6, pp. 843–854, Dec. 1979.
16. S. Mori, C.-Y. Chong, E. Tse, and R. Wishner
"Tracking and classifying multiple targets without a priori identification,"
IEEE Trans. Autom. Control, vol. 31, no. 5, pp. 401–409, May 1986.
17. S. S. Blackman
"Multiple hypothesis tracking for multiple target tracking,"
IEEE Aerosp. Electron. Syst. Mag., vol. 19, no. 1, pp. 5–18, Jan. 2004.
18. J. Roy, N. Duclos-Hindie, and D. Dessureault
"Efficient cluster management algorithm for multiple-hypothesis tracking,"
in *Proc. SPIE Signal Data Process. Small Targets*, 1997.

19. R. Danchick and G. E. Newnam
"A fast method for finding the exact N-best hypotheses for multitarget tracking,"
IEEE Trans. Aerosp. Electron. Syst., vol. 29, no. 2, pp. 555–560, Apr. 1993.
20. I. J. Cox and M. L. Miller
"On finding ranked assignments with application to multitarget tracking and motion correspondence,"
IEEE Trans. Aerosp. Electron. Syst., vol. 31, no. 1, pp. 486–489, Jan. 1995.
21. I. J. Cox, M. L. Miller, R. Danchick, and G. E. Newnam
"A comparison of two algorithms for determining ranked assignments with application to multitarget tracking and motion correspondence,"
IEEE Trans. Aerosp. Electron. Syst., vol. 33, no. 1, pp. 295–301, Jan. 1997.
22. K. G. Murty
"An algorithm for ranking all the assignments in order of increasing cost,"
Oper. Res., vol. 16, no. 3, pp. 682–687, 1968.
23. R. Danchick and G. E. Newnam
"Reformulating Reid's MHT method with generalised Murty K-best ranked linear assignment algorithm,"
IEE Proc.—Radar, Sonar Navig., vol. 153, no. 1, pp. 13–22, Feb. 2006.
24. E. Brekke and M. Chitre
"Success rates and posterior probabilities in multiple hypothesis tracking,"
in *Proc. 21st Int. Conf. Inf. Fusion*, Cambridge, U.K., 2018, pp. 949–956.
25. T. Kurien
"Issues in the design of practical multitarget tracking algorithms,"
in *Multitarget–Multisensor Tracking: Advanced Applications*, Y. Bar-Shalom
Ed. Norwood, MA: Artech House, 1989.
26. J. R. Delaney, A. S. Willsky, A. L. Blitz, and J. Korn
"Tracking theory for airborne surveillance radars,"
ALPHATECH, Burlington, MA, TR-142, Feb. 1983.
27. S. Mori and C.-Y. Chong
"Evaluation of data association hypotheses: non-Poisson i.i.d. cases,"
in *Proc. 7th Int. Conf. Inf. Fusion*, Stockholm, Sweden, 2004.
28. S. Coraluppi and C. A. Carthel
"If a tree falls in the woods, it does make a sound: Multiple-hypothesis tracking with undetected target births,"
IEEE Trans. Aerosp. Electron. Syst., vol. 50, no. 3, pp. 2379–2388, Jul. 2014.
29. Y. Bar-Shalom, S. S. Blackman, and R. J. Fitzgerald
"Dimensionless score function for multiple hypothesis tracking,"
IEEE Trans. Aerosp. Electron. Syst., vol. 43, no. 1, pp. 392–400, Jan. 2007.
30. S. Coraluppi, C. Carthel, M. Luettgen, and S. Lynch
"All-source track and identity fusion,"
in *Proc. MSS Nat. Symp. Sensor Data Fusion*, 2000.
31. A. Poore and N. Rijavec
"A Lagrangian relaxation algorithm for multidimensional assignment problems arising from multitarget tracking,"
SIAM J. Optim., vol. 3, no. 3, pp. 544–563, Aug. 1993.
32. A. B. Poore
"Multidimensional assignment formulation of data association problems arising from multitarget and multisensor tracking,"
Comput. Optim. Appl., vol. 3, no. 1, pp. 27–57, Mar. 1994.
33. S. Deb, M. Yeddanapudi, K. Pattipati, and Y. Bar-Shalom
"A generalized S-D assignment algorithm for multisensor–multitarget state estimation,"
IEEE Trans. Aerosp. Electron. Syst., vol. 33, no. 2, pp. 523–538, Apr. 1997.
34. D. J. Papageorgiou and M. R. Salpukas
"The maximum weight independent set problem for data association in multiple hypothesis tracking,"
in *Optimization and Cooperative Control Strategies*, M. J. Hirsch, C. W. Commander, P. M. Pardalos, and R. Murphey
Eds. Berlin, Germany: Springer, 2009, pp. 235–255.
35. A. Caponi
"Polynomial time algorithm for data association problem in multitarget tracking,"
IEEE Trans. Aerosp. Electron. Syst., vol. 40, no. 4, pp. 1398–1410, Oct. 2004.
36. X. Ren, Z. Huang, S. Sun, D. Liu, and J. Wu
"An efficient MHT implementation using GRASP,"
IEEE Trans. Aerosp. Electron. Syst., vol. 50, no. 1, pp. 86–101, Jan. 2014.
37. M. R. Chummun, T. Kirubarajan, K. R. Pattipati, and Y. Bar-Shalom
"Fast data association using multidimensional assignment with clustering,"
IEEE Trans. Aerosp. Electron. Syst., vol. 37, no. 3, pp. 898–913, Jul. 2001.
38. R. L. Popp, K. R. Pattipati, and Y. Bar-Shalom
"m-Best S-D assignment algorithm with application to multitarget tracking,"
IEEE Trans. Aerosp. Electron. Syst., vol. 37, no. 1, pp. 22–39, Jan. 2001.
39. E. Fortunato, W. Kreamer, S. Mori, C.-Y. Chong, and G. Castanon
"Generalized Murty's algorithm with application to multiple hypothesis tracking,"
in *Proc. 10th Int. Conf. Inf. Fusion*, Quebec City, QC, Canada, 2007.
40. C.-Y. Chong, S. Mori, and K.-C. Chang
"Distributed multitarget multisensor tracking,"
in *Multitarget–Multisensor Tracking: Advanced Applications*, Y. Bar-Shalom,
Ed. Norwood, MA: Artech House, 1989, pp. 247–296.
41. C.-Y. Chong and S. Mori
"Hierarchical multitarget tracking and classification—A Bayesian approach,"
in *Proc. Amer. Control Conf.*, 1984, pp. 599–604.
42. C.-Y. Chong, S. Mori, and K.-C. Chang
"Information fusion in distributed sensor networks,"
in *Proc. Amer. Control Conf.*, 1985, pp. 830–835.
43. M. E. Liggins, C.-Y. Chong, I. Kadar, M. G. Alford, V. Vannicola, and S. Thomopoulos
"Distributed fusion architectures and algorithms for target tracking,"
Proc. IEEE, vol. 85, no. 1, pp. 95–107, Jan. 1997.
44. C.-Y. Chong and S. Mori
"Optimal fusion for non-zero process noise,"
in *Proc. 16th Int. Conf. Inf. Fusion*, Istanbul, Turkey, 2013.
45. S. Coraluppi et al.
"Distributed MHT with active and passive sensors,"
in *Proc. 18th Int. Conf. Inf. Fusion*, Washington, DC, 2015.
46. D. T. Dunham, S. S. Blackman, and R. J. Dempster
"Multiple hypothesis tracking for a distributed multiple platform system,"
in *Proc. SPIE Signal Data Process. Small Targets*, 2004.
47. S. Coraluppi and C. Carthel
"Multi-stage multiple-hypothesis tracking,"
J. Adv. Inf. Fusion, vol. 6, no. 1, pp. 57–67, Jun. 2011.
48. O. E. Drummond
"Hybrid sensor fusion algorithm architecture and tracklets,"
in *Proc. SPIE Signal Data Process. Small Targets*, 1997.

49. C.-Y. Chong and S. Mori
"Multiple hypothesis tracking for processing tracklets,"
in *Proc. 22nd Int. Conf. Inf. Fusion*, Ottawa, Canada, 2019.
50. C.-Y. Chong, G. Castanon, N. Cooprider, S. Mori, R. Ravichandran, and R. Macior
"Efficient multiple hypothesis tracking by track segment graph,"
in *Proc. 12th Int. Conf. Inf. Fusion*, 2009, pp. 2177–2184.
51. C.-Y. Chong
"Graph approaches for data association,"
in *Proc. 15th Int. Conf. Inf. Fusion*, Singapore, 2012, pp. 1578–1585.
52. S. P. Coraluppi, C. A. Carthel, G. D. Castañón, and A. S. Willsky
"Multiple-hypothesis and graph-based tracking for kinematic and identity fusion,"
in *Proc. IEEE Aerosp. Conf.*, 2018.
53. J. K. Wolf, A. M. Viterbi, and G. S. Dixon
"Finding the best set of K paths through a trellis with application to multitarget tracking,"
IEEE Trans. Aerosp. Electron. Syst., vol. 25, no. 2, pp. 287–296, Mar. 1989.
54. D. A. Castanon
"Efficient algorithms for finding the K best paths through a trellis,"
IEEE Trans. Aerosp. Electron. Syst., vol. 26, no. 2, pp. 405–410, Mar. 1990.
55. G. Castanón and L. Finn
"Multi-target tracklet stitching through network flows,"
in *Proc. Aerosp. Conf.*, 2011.
56. R. K. Ahuja, T. L. Magnanti, and J. B. Orlin
Network Flows: Theory, Algorithms, and Applications. Upper Saddle River, NJ: Prentice Hall, 1993.
57. G. Battistelli, L. Chisci, F. Papi, A. Benavoli, A. Farina, and A. Graziano
"Optimal flow models for multiscan data association,"
IEEE Trans. Aerosp. Electron. Syst., vol. 47, no. 4, pp. 2405–2422, Oct. 2011.
58. L. Chen and S. E. Rumbley
"Track stitching and approximate track association on a pairwise-likelihood graph,"
in *Proc. SPIE Signal Process., Sensor/Inf. Fusion, Target Recognit. XXVII*, 2018.
59. B. F. La Scala
"Viterbi data association tracking using amplitude information,"
in *Proc. 6th Int. Conf. Inf. Fusion*, Stockholm, Sweden, 2004.
60. G. W. Pulford
"Multi-target Viterbi data association,"
in *Proc. 9th Int. Conf. Inf. Fusion*, Florence, Italy, 2006.
61. G. W. Pulford and B. F. La Scala
"Multihypothesis Viterbi data association: Algorithm development and assessment,"
IEEE Trans. Aerosp. Electron. Syst., vol. 46, no. 2, pp. 583–609, Apr. 2010.
62. L. van der Merwe and P. de Villiers
"Comparative investigation into Viterbi based and multiple hypothesis based track stitching,"
IET Radar, Sonar Navig., vol. 10, no. 9, pp. 1575–1582, 2016.
63. S. Mori and C.-Y. Chong
"Performance analysis of graph-based track stitching,"
in *Proc. 16th Int. Conf. Inf. Fusion*, Istanbul, Turkey, 2013, pp. 196–203.
64. L. Chen and R. Ravichandran
"A maximum weight constrained path cover algorithm for graph-based multitarget tracking,"
in *Proc. 18th Int. Conf. Inf. Fusion*, 2015, pp. 2073–2077.
65. L. Chen and R. Ravichandran
"On path cover problems with positive and negative constraints for graph-based multitarget tracking,"
in *Proc. 19th Int. Conf. Inf. Fusion*, Heidelberg, Germany, 2016, pp. 402–408.
66. S. Coraluppi, C. Carthel, W. Kreamer, and A. Willsky
"New graph-based and MCMC approaches to multi-INT surveillance,"
in *Proc. 19th Int. Conf. Inf. Fusion*, Istanbul, Turkey, 2016, pp. 394–401.
67. J. Fu, J. Sun, and P. Lei
"Graph-based tracklet stitching with feature information for ground target tracking,"
in *Geo-Spatial Knowledge and Intelligence*, H. Yuan, J. Geng, C. Liu, F. Bian, and T. Surapunt
Eds. Singapore: Springer, 2018, pp. 550–557.
68. S. P. Coraluppi, C. A. Carthel, and A. S. Willsky
"Multiple-hypothesis tracking and graph-based tracking extensions,"
J. Adv. Inf. Fusion, vol. 14, no. 2, Dec. 2019.
69. S. Mori, K.-C. Chang, and C.-Y. Chong
"Tracking aircraft by acoustic sensors: Multiple hypothesis approach applied to possibly unresolved measurements,"
in *Proc. Amer. Control Conf.*, Minneapolis, MN, 1987.
70. W. Koch and G. Van Keuk
"Multiple hypothesis track maintenance with possibly unresolved measurements,"
IEEE Trans. Aerosp. Electron. Syst., vol. 33, no. 3, pp. 883–892, Jul. 1997.
71. T. Sathyan, T. Chin, S. Arulampalam, and D. Suter
"A multiple hypothesis tracker for multitarget tracking with multiple simultaneous measurements,"
IEEE J. Sel. Top. Signal Process., vol. 7, no. 3, pp. 448–460, Jun. 2013.
72. S. P. Coraluppi and C. A. Carthel
"Multiple-hypothesis tracking for targets producing multiple measurements,"
IEEE Trans. Aerosp. Electron. Syst., vol. 54, no. 3, pp. 1485–1498, Jun. 2018.
73. K. Granstrom, M. Fatemi, and L. Svensson
"Poisson multi-Bernoulli conjugate prior for multiple extended object estimation," arXiv:1605.06311v6.
74. C. B. Storlie, T. C. M. Lee, J. Hannig, and D. Nychka
"Tracking of multiple merging and splitting targets: A statistical perspective,"
Stat. Sin., vol. 19, no. 1, pp. 1–52, Jan. 2009.
75. A. Makris and C. Prieur
"Bayesian multiple-hypothesis tracking of merging and splitting targets,"
IEEE Trans. Geosci. Remote Sens., vol. 52, no. 12, pp. 7684–7694, Dec. 2014.
76. R. L. Streit and T. E. Luginbuhl
"Probabilistic mutli-hypothesis tracking," Naval Undersea Warfare Center Div., Newport, RI, NUWC-NPT Tech. Rep. 10,428, Feb. 1995.
77. P. Willett, Y. Ruan, and R. Streit
"PMHT: Problems and some solutions,"
IEEE Trans. Aerosp. Electron. Syst., vol. 38, no. 3, pp. 738–754, Jul. 2002.
78. W. R. Gilks, S. Richardson, and D. Spiegelhalter
Markov Chain Monte Carlo in Practice. Boca Raton, FL: CRC Press, 1995.
79. M. Jerrum and A. Sinclair
"The Markov chain Monte Carlo method: An approach to approximate counting and integration,"
in *Approximation Algorithms for NP-Hard Problems*, D. S. Hochbaum
Ed. Boston, MA: PWS Publishing Co., 1997, pp. 482–520.
80. N. Bergman and A. Doucet
"Markov chain Monte Carlo data association for target tracking,"

in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, 2000.
81. S. Oh, S. Russell, and S. Sastry "Markov chain Monte Carlo data association for general multiple-target tracking problems," in *Proc. 43rd IEEE Conf. Decis. Control*, 2004.
82. S. Oh, S. Russell, and S. Sastry "Markov chain Monte Carlo data association for multi-target tracking," *IEEE Trans. Autom. Control*, vol. 54, no. 3, pp. 481–497, Mar. 2009.
83. S. Mori and C.-Y. Chong "Markov chain Monte Carlo method for evaluating multi-frame data association hypotheses," in *Proc. 11th Int. Conf. Inf. Fusion*, Cologne, Germany, 2008.
84. S. Mori and C. Chong "Cross-entropy method for *K*-best dependent-target data association hypothesis selection," in *Proc. 13th Int. Conf. Inf. Fusion*, Edinburgh, U.K., 2010.
85. M. I. Jordan "Graphical models," *Stat. Sci.*, vol. 19, no. 1, pp. 140–155, Feb. 2004.
86. F. Meyer et al. "Message passing algorithms for scalable multitarget tracking," *Proc. IEEE*, vol. 106, no. 2, pp. 221–259, Feb. 2018.
87. A. Frank, P. Smyth, and A. Ihler "Beyond MAP estimation with the track-oriented multiple hypothesis tracker," *IEEE Trans. Signal Process.*, vol. 62, no. 9, pp. 2413–2423, May 2014.
88. Q. Li, J. Sun, and W. Sun "An efficient multiple hypothesis tracker using max product belief propagation," in *Proc. 20th Int. Conf. Inf. Fusion*, Xi'an, China, 2017.
89. W. Koch "Retrodiction for Bayesian multiple-hypothesis/multiple-target tracking in densely cluttered environment," in *Proc. SPIE Signal Data Process. Small Targets*, 1996.
90. W. Koch "Fixed-interval retrodiction approach to Bayesian IMM-MHT for maneuvering multiple targets," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 36, no. 1, pp. 2–14, Jan. 2000.
91. D. F. Crouse, P. Willett, and Y. Bar-Shalom "Developing a real-time track display that operators do not hate," *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3441–3447, Jul. 2011.
92. D. Schuhmacher, B. Vo, and B. Vo "A consistent metric for performance evaluation of multi-object filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3447–3457, Aug. 2008.
93. K.-C. Chang, S. Mori, and C.-Y. Chong "Performance evaluation of a multiple-hypothesis multi-target tracking algorithm," in *Proc. 29th IEEE Conf. Decis. Control*, 1990.
94. K.-C. Chang, S. Mori, and C.-Y. Chong "Evaluating a multiple-hypothesis multitarget tracking algorithm," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 30, no. 2, pp. 578–590, Apr. 1994.
95. S. D. Blostein and T. S. Huang "Detecting small, moving objects in image sequences using sequential hypothesis testing," *IEEE Trans. Signal Process.*, vol. 39, no. 7, pp. 1611–1629, Jul. 1991.
96. A. Wald *Sequential Analysis*. New York, NY: Wiley, 1947.
97. G. V. Keuk "Sequential track extraction," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, no. 4, pp. 1135–1148, Oct. 1998.
98. E. Wilthil, E. Brekke, and O. Asplin "Track initiation for maritime radar tracking with and without prior information," in *Proc. 21st Int. Conf. Inf. Fusion*, Cambridge, U.K., 2018.
99. R. P. S. Mahler *Statistical Multisource–Multitarget Information Fusion*. Norwood, MA: Artech House, 2007.
100. R. P. S. Mahler *Advances in Statistical Multisource--Multitarget Information Fusion*. Norwood, MA: Artech House, 2014.
101. R. P. S. Mahler "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.
102. Á. F. García-Fernández and B. Vo "Derivation of the PHD and CPHD filters based on direct Kullback–Leibler divergence minimization," *IEEE Trans. Signal Process.*, vol. 63, no. 21, pp. 5812–5820, Nov. 2015.
103. J. L. Williams "An efficient, variational approximation of the best fitting multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 63, no. 1, pp. 258–273, Jan. 2015.
104. M. Ulmke, O. Erdinc, and P. Willett "Gaussian mixture cardinalized PHD filter for ground moving target tracking," in *Proc. 10th Int. Conf. Inf. Fusion*, Quebec City, QC, Canada, 2007.
105. J. L. Williams "Marginal multi-Bernoulli filters: RFS derivation of MHT, JIPDA, and association-based member," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 3, pp. 1664–1687, Jul. 2015.
106. E. F. Brekke and M. Chitre "The multiple hypothesis tracker derived from finite set statistics," in *Proc. 20th Int. Conf. Inf. Fusion*, Xi'an, China, 2017.
107. E. Brekke and M. Chitre "Relationship between finite set statistics and the multiple hypothesis tracker," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 4, pp. 1902–1917, Aug. 2018.
108. Á. F. García-Fernández, J. L. Williams, K. Granström, and L. Svensson "Poisson multi-Bernoulli mixture filter: Direct derivation and implementation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 4, pp. 1883–1901, Aug. 2018.
109. L. Svensson and M. Morelande "Target tracking based on estimation of sets of trajectories," in *Proc. 17th Int. Conf. Inf. Fusion*, Salamanca, Spain, 2014.
110. K. Granström, L. Svensson, Y. Xia, J. Williams, and A. F. Garcia-Fernandez "Poisson multi-Bernoulli mixture trackers: Continuity through random finite sets of trajectories," in *Proc. 21st Conf. Inf. Fusion*, Cambridge, U.K., 2018.
111. Y. Xia, K. Granström, L. Svensson, and Á. F. García-Fernández "An implementation of the Poisson multi-Bernoulli mixture trajectory filter via dual decomposition," in *Proc. 21st Int. Conf. Inf. Fusion*, Cambridge, U.K., 2018.

112. Y. Xia, K. Granström, L. Svensson, A. F. Garcia-Fernandez, and J. L. Williams
"Multi-scan implementation of the trajectory Poisson multi-Bernoulli mixture filter,"
J. Adv. Inf. Fusion, vol. 14, no. 2, Dec. 2019.
113. B.-T. Vo and B.-N. Vo
"Labeled random finite sets and multi-object conjugate priors,"
IEEE Trans. Signal Process., vol. 61, no. 13, pp. 3460–3475, Jul. 2013.
114. B.-N. Vo, B.-T. Vo, and D. Phung
"Labeled random finite sets and the Bayes multi-target tracking filter,"
IEEE Trans. Signal Process., vol. 62, no. 24, pp. 6554–6567, Dec. 2014.
115. S. Mori, C.-Y. Chong, and K.-C. Chang
"Three formalisms of multiple hypothesis tracking,"
in *Proc. 19th Int. Conf. Inf. Fusion*, Heidelberg, Germany, 2016, pp. 727–734.
116. S. Mori, C.-Y. Chong, and K.-C. Chang
"Three mathematical formalisms of multiple hypothesis tracking,"
J. Adv. Inf. Fusion, vol. 14, no. 2, Dec. 2019.
117. H. L. Wiener, W. W. Willman, J. H. Kullback, and I. R. Goodman
"Naval Ocean-Surveillance Correlation Handbook, 1978,"
Naval Res. Lab., Washington, DC, NRL Rep. 8340, Oct. 1979.
118. D. B. Reid
"The application of multiple target tracking theory to ocean surveillance,"
in *Proc. 18th IEEE Conf. Decis. Control*, 1979.
119. C. L. Bowman and M. S. Murphy
"An architecture for fusion of multisensor ocean surveillance data,"
in *Proc. 20th IEEE Conf. Decis. Control*, 1981.
120. C. M. Smith and J. J. Leonard
"A multiple-hypothesis approach to concurrent mapping and localization for autonomous underwater vehicles,"
in *Field and Service Robotics*, A. Zelinsky
Ed. London, U.K.: Springer, 1998, pp. 237–244.
121. S. Coraluppi and C. Carthel
"Multi-hypothesis sonar tracking,"
in *Proc. 4th Int. Conf. Inf. Fusion*, Montreal, Canada, 2001.
122. S. Coraluppi and C. Carthel
"Distributed tracking in multistatic sonar,"
IEEE Trans. Aerosp. Electron. Syst., vol. 41, no. 3, pp. 1138–1147, Jul. 2005.
123. S. Coraluppi and C. Carthel
"Advances in active sonar tracking,"
in *Proc. 14th Eur. Signal Process. Conf.*, 2006.
124. S. Coraluppi, C. Carthel, and A. Coon
"An MHT approach to multi-sensor passive sonar tracking,"
in *Proc. 21st Int. Conf. Inf. Fusion*, Cambridge, U.K., 2018.
125. M. Balci and R. Pegg
"Towards global maritime domain awareness—'Recent developments and challenges',"
in *Proc. 9th Int. Conf. Inf. Fusion*, Florence, Italy, 2006.
126. N. A. Bomberger, B. J. Rhodes, M. Seibert, and A. M. Waxman
"Associative learning of vessel motion patterns for maritime situation awareness,"
in *Proc. 9th Int. Conf. Inf. Fusion*, Florence, Italy, 2006.
127. M. Seibert et al.
"SeeCoast port surveillance,"
in *Proc. Photon. Port Harbor Secur. II*, 2006, vol. 6204.
128. C. Carthel, S. Coraluppi, and P. Grignan
"Multisensor tracking and fusion for maritime surveillance,"
in *Proc. 10th Int. Conf. Inf. Fusion*, Quebec City, QC, Canada, 2007.
129. E. Liland
"AIS aided multi hypothesis tracker," Master's thesis, Norwegian Univ. Sci. Technol., Trondheim, Norway, 2017.
130. O. Gerard, C. Carthel, and S. Coraluppi
"Estimating the number of beaked whales using an MHT tracker,"
in *Proc. 2008 New Trends Environ. Monit. Passive Syst.*, Hyeres, French Riviera, France, 2008, pp. 1–6.
131. J. N. Entzminger, C. A. Fowler, and W. J. Kenneally
"JointSTARS and GMTI: Past, present and future,"
IEEE Trans. Aerosp. Electron. Syst., vol. 35, no. 2, pp. 748–761, Apr. 1999.
132. C.-Y. Chong, D. Garren, and T. P. Grayson
"Ground target tracking—A historical perspective,"
in *Proc. IEEE Aerosp. Conf.*, 2000.
133. D. R. Kirk, T. Grayson, D. Garren, and C.-Y. Chong
"AMSTE precision fire control tracking overview,"
in *Proc. IEEE Aerosp. Conf.*, 2000.
134. P. J. Shea, T. Zadra, D. Klamer, E. Frangione, R. Brouillard, and K. Kastella
"Precision tracking of ground targets,"
in *Proc. IEEE Aerosp. Conf.*, 2000.
135. R. J. Dempster, S. S. Blackman, and T. S. Nichols
"Combining IMM filtering and MHT data association for multitarget tracking,"
in *Proc. 29th Southeastern Symp. Syst. Theory*, 1997.
136. W. Koch
"GMTI-tracking and information fusion for ground surveillance,"
in *Proc. SPIE Signal Data Process. Small Targets*, 2001.
137. J. Koller and M. Ulmke
"Multi hypothesis track extraction and maintenance of GMTI sensor data,"
in *Proc. 7th Int. Conf. Inf. Fusion*, Philadelphia, PA, 2005.
138. D. Svensson, J. Wintenby, and L. Svensson
"Performance evaluation of MHT and GM-CPHD in a ground target tracking scenario,"
in *Proc. 12th Int. Conf. Inf. Fusion*, Seattle, WA, 2009.
139. P. O. Arambel, J. Silver, J. Krant, M. Antone, and T. Strat
"Multiple-hypothesis tracking of multiple ground targets from aerial video with dynamic sensor control,"
in *Proc. SPIE Signal Process., Sensor Fusion, Target Recognit. XIII*, 2004.
140. P. O. Arambel and M. Antone
"Multiple target tracking with a steerable airborne video sensor,"
in *Proc. SPIE Signal Process., Sensor Fusion, Target Recognit. XV*, 2006.
141. M. Keck, L. Galup, and C. Stauffer
"Real-time tracking of low-resolution vehicles for wide-area persistent surveillance,"
in *Proc. 2013 IEEE Workshop Appl. Comput. Vis.*, 2013.
142. P. Bendich et al.
"Topological and statistical behavior classifiers for tracking applications,"
IEEE Trans. Aerosp. Electron. Syst., vol. 52, no. 6, pp. 2644–2661, Dec. 2016.
143. R. Spraul, C. Hartung, and T. Schuchert
"Persistent multiple hypothesis tracking for wide area motion imagery,"
in *Proc. 2017 IEEE Int. Conf. Image Process.*, 2017.
144. B. Uzkent, M. J. Hoffman, and A. Vodacek
"Integrating hyperspectral likelihoods in a multidimensional assignment algorithm for aerial vehicle tracking,"
IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens., vol. 9, no. 9, pp. 4325–4333, Sep. 2016.

145. A. G. A. Perera, C. Srinivas, A. Hoogs, G. Brooksby, and W. Hu "Multi-object tracking through simultaneous long occlusions and split–merge conditions," in *Proc. Conf. Comput. Vis. Pattern Recognit.*, 2006.
146. Y. Z. Gürbüz, O. Can, and A. A. Alatan "Roadesic distance: Flow-aware tracklet association cost for wide area surveillance," in *Proc. IEEE Int. Conf. Image Process.*, 2017.
147. D. E. Iverson, K.-C. Chang, and C.-Y. Chong "Performance assessment for airborne surveillance systems incorporating sensor fusion," in *Proc. SPIE Signal Data Process. Small Targets*, 1989.
148. S. S. Blackman, R. J. Dempster, and T. J. Broida "Multiple hypothesis track confirmation for infrared surveillance systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, no. 3, pp. 810–824, Jul. 1993.
149. G. Van Keuk "Multihypothesis tracking with electronically scanned radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 31, no. 3, pp. 916–927, Jul. 1995.
150. T. Wigren, E. Sviestins, and H. Egnell "Operational multi-sensor tracking for air defense," in *Proc. 1st Aust. Data Fusion Symp.*, 1996.
151. S. S. Blackman, R. J. Dempster, M. T. Busch, and R. F. Popoli "IMM/MHT solution to radar benchmark tracking problem," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 35, no. 2, pp. 730–738, Apr. 1999.
152. W. Koch "Generalized smoothing for multiple model/multiple hypothesis filtering: Experimental results," in *Proc. Eur. Control Conf.*, 1999.
153. G. Van Keuk "MHT extraction and track maintenance of a target formation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 38, no. 1, pp. 288–295, Jan. 2002.
154. A. Thite and A. Mishra "Optimized multi-sensor multi-target tracking algorithm for air surveillance system," in *Proc. 2nd Int. Conf. Adv. Elect., Electron., Inf., Commun. Bioinf.*, 2016.
155. R. A. Hoogendoorn and W. H. L. Neven "ARTAS: Multisensor tracking in an ATC environment," Nat. Aerosp. Lab., Emmeloord, The Netherlands, NLR TP 97657, 1997.
156. B. Rakdham, M. Tummala, P. E. Pace, J. B. Michael, and Z. P. Pace "Boost phase ballistic missile defense using multiple hypothesis tracking," in *Proc. IEEE Int. Conf. Syst. Syst. Eng.*, 2007.
157. J. A. Kennewell and B.-N. Vo "An overview of space situational awareness," in *Proc. 16th Int. Conf. Inf. Fusion*, Istanbul, Turkey, 2013.
158. A. B. Poore and J. T. Horwood "Multiple hypothesis correlation for space situational awareness," Numerica Corp., Loveland, CO, Final Tech. Rep., Aug. 2011.
159. S. M. Gadaleta, J. T. Horwood, and A. B. Poore "Short arc gating in multiple hypothesis tracking for space surveillance," in *Proc. SPIE Sensors Syst. Space Appl. V*, 2012.
160. N. Singh, J. T. Horwood, J. M. Aristoff, A. B. Poore, C. Sheaff, and M. K. Jah "Multiple hypothesis tracking (MHT) for space surveillance: Results and simulation studies," in *Proc. Adv. Maui Opt. Space Surveillance Technol. Conf.*, 2013.
161. I. J. Cox, J. M. Rehg, and S. Hingorani "A Bayesian multiple hypothesis approach to contour grouping," in *Proc. Eur. Conf. Comput. Vis.*, 1992.
162. I. J. Cox, J. M. Rehg, and S. Hingorani "A Bayesian multiple-hypothesis approach to edge grouping and contour segmentation," *Int. J. Comput. Vis.*, vol. 11, no. 1, pp. 5–24, Aug. 1993.
163. I. J. Cox and S. L. Hingorani "An efficient implementation and evaluation of Reid's multiple hypothesis tracking algorithm for visual tracking," in *Proc. 12th Int. Conf. Pattern Recognit.*, 1994.
164. T.-J. Cham and J. M. Rehg "A multiple hypothesis approach to figure tracking," in *Proc. Conf. Comput. Vis. Pattern Recognit.*, 1999.
165. C. Stauffer "Estimating tracking sources and sinks," in *Proc. Conf. Comput. Vis. Pattern Recognit.*, 2003.
166. A. Y. S. Chia, W. Huang, and L. Li "Multiple objects tracking with multiple hypotheses graph representation," in *Proc. 18th Int. Conf. Pattern Recognit.*, 2006.
167. H. Jiang, S. Fels, and J. J. Little "A linear programming approach for multiple object tracking," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2007.
168. V. K. Singh, B. Wu, and R. Nevatia "Pedestrian tracking by associating tracklets using detection residuals," in *Proc. IEEE Workshop Motion Video Comput.*, 2008.
169. K. O. Arras, S. Grzonka, M. Luber, and W. Burgard "Efficient people tracking in laser range data using a multi-hypothesis leg-tracker with adaptive occlusion probabilities," in *Proc. IEEE Int. Conf. Robot. Autom.*, 2008.
170. L. Zhang, Y. Li, and R. Nevatia "Global data association for multi-object tracking using network flows," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2008.
171. J. Xing, H. Ai, and S. Lao "Multi-object tracking through occlusions by local tracklets filtering and global tracklets association with detection responses," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2009.
172. Z. Wu, N. I. Hristov, T. H. Kunz, and M. Betke "Tracking-reconstruction or reconstruction-tracking? Comparison of two multiple hypothesis tracking approaches to interpret 3D object motion from several camera views," in *Proc. Workshop Motion Video Comput.*, 2009.
173. A. Azim and O. Aycard "Multiple pedestrian tracking using Viterbi data association," in *Proc. IEEE Intell. Veh. Symp.*, 2010.
174. X. Yu and A. Ganz "Global identification of tracklets in video using long range identity sensors," in *Proc. 7th IEEE Int. Conf. Adv. Video Signal Based Surveillance*, 2010.
175. J. Berclaz, F. Fleuret, E. Turetken, and P. Fua "Multiple object tracking using *K*-shortest paths optimization," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 33, no. 9, pp. 1806–1819, Sep. 2011.

176. H. Pirsiavash, D. Ramanan, and C. C. Fowlkes
"Globally-optimal greedy algorithms for tracking a variable number of objects,"
in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2011.
177. W. Brendel, M. Amer, and S. Todorovic
"Multiobject tracking as maximum weight independent set,"
in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2011.
178. Z. Wu, T. H. Kunz, and M. Betke
"Efficient track linking methods for track graphs using network-flow and set-cover techniques,"
in *Proc. Conf. Comput. Vis. Pattern Recognit.*, 2011.
179. L. Lin, Y. Lu, Y. Pan, and X. Chen
"Integrating graph partitioning and matching for trajectory analysis in video surveillance,"
IEEE Trans. Image Process., vol. 21, no. 12, pp. 4844–4857, Dec. 2012.
180. A. A. Butt and R. T. Collins
"Multi-target tracking by Lagrangian relaxation to min-cost network flow,"
in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2013.
181. H. B. Shitrit, J. Berclaz, F. Fleuret, and P. Fua
"Multi-commodity network flow for tracking multiple people,"
IEEE Trans. Pattern Anal. Mach. Intell., vol. 36, no. 8, pp. 1614–1627, Aug. 2014.
182. T. Linder and K. O. Arras
"Multi-model hypothesis tracking of groups of people in RGB-D data,"
in *Proc. 17th Int. Conf. Inf. Fusion*, Salamanca, Spain, 2014.
183. C. Kim, F. Li, A. Ciptadi, and J. M. Rehg
"Multiple hypothesis tracking revisited,"
in *Proc. IEEE Int. Conf. Comput. Vis.*, 2015.
184. B. Wang, G. Wang, K. L. Chan, and L. Wang
"Tracklet association by online target-specific metric learning and coherent dynamics estimation,"
IEEE Trans. Pattern Anal. Mach. Intell., vol. 39, no. 3, pp. 589–602, Mar. 2017.
185. M. Manafifard, H. Ebadi, and H. A. Moghaddam
"Appearance-based multiple hypothesis tracking,"
Image Commun., vol. 55, no. C, pp. 157–170, Jul. 2017.
186. O. Birbach and U. Frese
"A multiple hypothesis approach for a ball tracking system,"
in *Computer Vision Systems*, M. Fritz, B. Schiele, and J. H. Piater
Eds. Berlin, Germany: Springer, 2009, pp. 435–444.
187. X. Zhou, L. Xie, Q. Huang, S. J. Cox, and Y. Zhang
"Tennis ball tracking using a two-layered data association approach,"
IEEE Trans. Multimedia, vol. 17, no. 2, pp. 145–156, Feb. 2015.
188. "MOT challenge." [Online]. Accessed: Feb. 15, 2019. Available: https://motchallenge.net/.
189. J. H. Faghmous et al.
"Multiple hypothesis object tracking for unsupervised self-learning: An ocean eddy tracking application,"
in *Proc. 23th AAAI Conf. Artif. Intell.*, 2013.
190. J. Kubica, A. Moore, A. Connolly, and R. Jedicke
"A multiple tree algorithm for the efficient association of asteroid observations,"
in *Proc. 11th ACM SIGKDD Int. Conf. Knowl. Discov. Data Mining*, New York, NY, 2005.
191. D. Kempton, K. G. Pillai, and R. Angryk
"Iterative refinement of multiple targets tracking of solar events,"
in *Proc. 2014 IEEE Int. Conf. Big Data*, Washington, DC, 2014.
192. D. J. Kempton and R. A. Angryk
"Tracking solar events through iterative refinement,"
Astron. Comput., vol. 13, pp. 124–135, Nov. 2015.
193. G. V. Cybenko and C. Alberola
"Tracking with text-based messages,"
IEEE Intell. Syst., vol. 14, no. 4, pp. 70–78, 1999.
194. S. Aja-Fernández, C. Alberola-López, and G. Cybenko
"A fuzzy MHT algorithm applied to text-based information tracking,"
IEEE Trans. Fuzzy Syst., vol. 10, pp. 360–374, Jul. 2002.
195. V. H. Berk, G. V. Cybenko, and R. S. Gray
"Early detection of active Internet worms,"
in *Managing Cyber Threats: Issues, Approaches, and Challenges*, V. Kumar, J. Srivastava, and A. Lazarevic
Eds. Boston, MA: Springer US, 2005, pp. 147–180.
196. S. Schwoegler, J. Holsopple, S. Blackman, and M. J. Hirsch
"On the application of multiple hypothesis tracking to the cyber domain,"
in *Proc. 14th Int. Conf. Inf. Fusion*, Chicago, IL, 2011.
197. J. H. P. Broeke
et al., "Automated quantification of cellular traffic in living cells,"
J. Neurosci. Methods, vol. 178, no. 2, pp. 378–384, Apr. 2009.
198. L. Feng, Y. Xu, Y. Yang, and X. Zheng
"Multiple dense particle tracking in fluorescence microscopy images based on multidimensional assignment,"
J. Struct. Biol., vol. 173, no. 2, pp. 219–228, Feb. 2011.
199. N. Chenouard, I. Bloch, and J. Olivo-Marin
"Multiple hypothesis tracking in microscopy images,"
in *Proc. 2009 IEEE Int. Symp. Biomed. Imag.: From Nano to Macro*, 2009.
200. N. Chenouard, I. Bloch, and J. Olivo-Marin
"Multiple hypothesis tracking for cluttered biological image sequences,"
IEEE Trans. Pattern Anal. Mach. Intell., vol. 35, no. 11, pp. 2736–3750, Nov. 2013.
201. E. Cheng, Y. Pang, Y. Zhu, J. Yu, and H. Ling
"Curvilinear structure tracking by low rank tensor approximation with model propagation,"
in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, 2014.
202. L. Liang, H. Shen, P. De Camilli, and J. S. Duncan
"A novel multiple hypothesis based particle tracking method for clathrin mediated endocytosis analysis using fluorescence microscopy,"
IEEE Trans. Image Process., vol. 23, no. 4, pp. 1844–1857, Apr. 2014.

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